

Technical Note Describing New Lake Model for CLM4

Zachary M. Subin and William J. Riley
Lawrence Berkeley National Laboratory
University of California, Berkeley

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1 Overview

This document describes the new CLM 4 lake model as discussed in the accompanying manuscript (Subin, Z.M., W.J. Riley, and D. Mironov, 2011. An Improved Lake Model for Regional and Global Climate Simulations: Development, Evaluation, and Sensitivity Analyses in CESM1. J. Adv. Mod. Earth Sys.) See the manuscript for references and discussion.

The new lake model contains the following principle modifications compared to the existing CLM4 (which is detailed in Oleson et al. (2010)):

1. For snow greater than a minimum depth, the full CLM4 snow model is used, rather than the simple bulk snow scheme without thermal insulation used in existing CLM4 lakes.

2. Phase change and ice physics are included in the lake body.
3. A sediment thermal submodel is coupled beneath the lake body, based on the CLM4 soil and bedrock with assumed hydraulic saturation.
4. Lakes have variable depth. The default is retained to be 50 m if there is no explicit depth in the surface input dataset. However, Kourzeneva et al. (2010) provides a lake depth dataset at 1 km.
5. Parameterizations of roughness lengths, albedo, and extinction coefficient are improved.
6. Enhanced diffusion is added according to Fang and Stefan (1996) to parameterize mixing due to unresolved 3D processes, with an option for increasing overall mixing by an additional factor for deep lakes.
7. Several errors in the calculation of surface temperature, surface fluxes, and eddy diffusivity are corrected, with large improvements in lake behavior compared with observations.

The following sections detail the equations used in the model code, roughly in the order in which they are called by the CLM driver. When the code follows closely with the original CLM4 code, only the modified portions are listed.

Note the following conventions. “Ground” refers to the earth surface (e.g. snow, ice, or lake water) interface with the atmosphere. This interface has a well-defined temperature T_g but is associated with zero heat capacity. If not otherwise specified, “CLM4” refers to the existing CLM4 model (including lake submodel), as opposed to the new lake model described here. This model is documented in Oleson et al. (2010), http://www.cesm.ucar.edu/models/ccsm4.0/clm/CLM4_Tech_Note.pdf

2 Discretization

As currently implemented, the lake consists of 0-5 snow layers, water and ice layers (10 for global simulations and 25 for site simulations) comprising the lake body, 10 “soil” layers, and 5 “bedrock” layers. Resolved snow layers are present if the snow thickness z_{sno} is greater than a minimum thickness $s_{min} = 4$ cm. (This is greater than the minimum thickness of 1 cm over other landunits in CLM4; see 6.1 and 6.4.) For global simulations with 10 body layers, the thicknesses are the same as in CLM4 when the lake is 50 m. Otherwise, the top layer is kept at 10 cm and the other 9 layer thicknesses are adjusted proportionally, except for lakes of depth less than 1 m, in which case all layers have equal thickness. For site simulations with 25 layers, the thicknesses are (m): 0.1 for layer 1; 0.25 for layers 2-5; 0.5 for layers 6-9; 0.75 for layers 10-13; 2 for layers 14-15; 2.5 for layers 16-17; 3.5 for layers 18-21; and 5.225 for layers 22-25. Soil and snow thicknesses are the same as in CLM4.

3 Surface Albedo

For direct radiation, the albedo a for lakes with ground temperature T_g above freezing is given by (Pivovarov, 1972)

$$a = \frac{0.05}{\cos z + 0.15}, \quad (1)$$

where z is the zenith angle in radians, $z \in (0, \frac{\pi}{2})$. For diffuse radiation, the expression in Eq. 1 is integrated over the full sky to yield $a = 0.10$.

For frozen lakes without resolved snow layers, the albedo at cold temperatures a_0 is 0.60 for visible and 0.40 for near infrared radiation. As the temperature at the ice surface, T_g , approaches freezing (T_f), the albedo is relaxed towards 0.10 based on Mironov (2010b):

$$a = a_0(1 - x) + 0.10x, x = \exp\left(-95\frac{T_f - T_g}{T_f}\right), \quad (2)$$

where temperatures are in K, and a is restricted to be no less than the a given by Eq. 1.

For frozen lakes with resolved snow layers, the albedo is predicted by the CLM4 snow-optics submodel.

4 Surface Flux Solution

4.1 Review of Changes from CLM4 Surface Flux Solution

The lake biogeophysics has been split for shallow lakes into two modules, “SLakeFluxes” and “SLakeTemperature”.

In “SLakeFluxes”, the solution of the surface fluxes retains the same broad form as in the CLM4 “BiogeophysicsLake” module. The following modifications are made:

1. The surface roughnesses z_{0m} , z_{0q} , and z_{0h} , and surface shortwave absorption fraction β have been modified; see Section 4.2 below. (The emissivity remains 0.97.)
2. Bug fix: the original lake code solved for the ground temperature over each iteration using Newton’s Method as per the following equation:

$$T_g^{n+1} = \frac{S_g - L_g - H_g - \lambda E_g - G + T_g^n \left(\frac{\partial L_g}{\partial T_g} + \frac{\partial H_g}{\partial T_g} + \frac{\partial \lambda E_g}{\partial T_g} + \frac{\partial G}{\partial T_g} \right)}{\frac{\partial L_g}{\partial T_g} + \frac{\partial H_g}{\partial T_g} + \frac{\partial \lambda E_g}{\partial T_g} + \frac{\partial G}{\partial T_g}} \quad (3)$$

where T_g is the ground temperature, T_g^n is the previous iteration ground temperature, T_g^{n+1} is the ground temperature for the current iteration, S_g is the net shortwave flux, L_g is the net longwave flux, H_g is the sensible heat flux, λE_g is the latent heat flux, and G is the ground heat flux.

This is incorrect because the fraction of solar radiation absorbed at the ground “surface” (an infinitesimal interface) is defined elsewhere to be the fraction βS_g . Indeed, the flux “fin” into the lake in the BiogeophysicsLake module uses this fraction. It subtracts the actual surface fluxes (e.g. H_g) from the flux into the lake in this module, so energy is conserved, but the sum $H_g + L_g + \lambda E_g$ has been effectively increased by about $(1 - \beta S_g)$, while the flux into the lake “fin” has been reduced by the same quantity. In the SLakeFluxes module, the correct expression is used.

3. Bug fix: the thickness of the top layer is used in the calculation of T_g via the equation

$$\frac{\partial G}{\partial T_g} = \frac{\lambda_{j_{top}}}{\Delta z_{j_{top}}} \quad (4)$$

where $\lambda_{j_{top}}$ is the thermal conductivity of the top layer, and $\Delta z_{j_{top}}$ is its thickness. However, since the temperature of the top layer is defined at its node (midpoint for lake or snow layers), this should be $\frac{1}{2}\Delta z_{j_{top}}$. This is used in “SLakeFluxes”.

4. Top layer: the temperature and thermal properties of the top layer are used in the calculation of the ground temperature. In SLakeFluxes, the top layer is either a lake layer or a snow layer, and the temperature of the respective layer is used. For the thermal conductivity, there are three cases.

- (a) If the top layer is completely thawed and there are no resolved snow layers, and the ground temperature from the previous iteration is above freezing, then an effective eddy thermal conductivity is used using the eddy diffusion coefficient from the previous timestep for the top lake layer, e.g.

$$\lambda_1 = \kappa_{e,1} \rho_{liq} c_{liq} + \lambda_m \quad (5)$$

where λ_1 is the desired thermal conductivity, $\kappa_{e,1}$ is the eddy diffusivity as defined in Section 5.3, ρ_{liq} is the density of water, c_{liq} is the specific heat of water, and λ_m is the molecular thermal conductivity of water. In CLM4, only the molecular thermal conductivity was used, but this is a severe underestimation of the mixing in the top 0.1-1 m of the lake.

- (b) If the ground temperature or that of the top lake layer is at or below freezing, but there are no snow layers, then the thermal conductivity of ice is used.
 - (c) If the top layer is a snow layer, then the actual thermal conductivity of that layer is used as defined in CLM4.
5. Re-evaluation of ground temperature: there are three cases where the ground temperature is changed and the surface fluxes re-evaluated before being finalized.

- (a) Snow is present or the top lake layer is at or below freezing, and the ground temperature is above freezing. In this case, the ground temperature is reset to freezing.
- (b) The ground is cooler than the top lake layer but warmer than 4°C.
- (c) The ground is less than 4°C but warmer than the lake, which is warmer than freezing.

In the latter two cases, the lake surface will be more dense than the top lake layer, so mixing would occur until the skin equals the ground temperature. Thus, the ground temperature is set to be equal to the top lake layer temperature.

After the ground temperature is changed, the surface fluxes are re-evaluated with respect to the new ground temperature.

For the surface evaporation, like in CLM4, the function “QSat” is not called again, but rather the previously calculated derivative $\frac{dq_{sat}}{dT_g}$ is used to reconcile the saturation vapor pressure q_{sat} between the previously calculated ground temperature (when “QSat” was last called) and the new ground temperature: $q_{sat}(T_g) = q_{sat}(T_{g,bef}) + \frac{dq_{sat}}{dT_g}(T_{g,bef}) \times (T_g - T_{g,bef})$. This is a bug fix, in that in CLM4 the ground temperature *from the previous iteration* was used, not the ground temperature from the most recent iteration but before the correction for the three cases listed here. A series of comments in the code indicated that this discrepancy was noted before but not corrected.

- 6. Iteration for ground temperature: 4 iterations are now used to allow extra time for convergence since the friction velocity u_* and surface roughnesses are now functions of each other.

4.2 Surface Properties

The surface roughnesses are now functions of the lake state and atmospheric forcing. For frozen lakes ($T_g \leq T_f$) with resolved snow layers, the momentum roughness length $z_{0m} = 2.4 \times 10^{-3}$ m, and

$$z_{0q} = z_{0h} = z_{0m} \exp \left\{ -0.13 \left(\frac{u_* z_{0m}}{1.5 \times 10^{-5}} \right)^{0.45} \right\}, \quad (6)$$

where z_{0h} (m) is the surface roughness for sensible heat and z_{0q} (m) is the surface roughness for latent heat, and the latter expression should be generalized to be a function of temperature, but has been kept consistent with the expressions in “BareGroundFluxes” in CLM4. For frozen lakes with no resolved snow layers, $z_{0m} = 1 \times 10^{-3}$ m, and the scalar roughnesses are given by Eq. 6.

For unfrozen snow layers,

$$z_{0m} = \max \left(\frac{\alpha \nu}{u_*}, \frac{c u_*^2}{g} \right), \quad (7)$$

where α is 0.1, ν is the kinematic viscosity of air given below, c is the Charnock coefficient given below, and g is the acceleration of gravity.

$$\nu = \nu_0 \left(\frac{T_g}{T_0} \right)^{1.5} \frac{P_0}{P_{ref}}, \quad (8)$$

where $\nu_0 = 1.51 \times 10^{-5} \frac{\text{m}^2}{\text{s}}$ is the kinematic viscosity at $T_0 = 293.15\text{K}$ and $P_0 = 1.013 \times 10^5 \text{ Pa}$, and P_{ref} is the pressure at the atmospheric reference height. The Charnock coefficient is a function of the lake fetch F (m), given in external data or set to 25 times the lake depth d if such data is unavailable:

$$c = c_0 + c_{max} \exp \{ \min(A, B) \}, \quad (9)$$

$$A = \frac{\left(\frac{Fg}{u^2} \right)^{\frac{1}{3}}}{f_c}, \quad (10)$$

$$B = \frac{\sqrt{dg}}{u}. \quad (11)$$

$c_0 = 0.01$, $c_{max} = 0.1$, u is the forcing wind, and $f_c = 22$ is the critical dimensionless fetch. The scalar roughness lengths are given by

$$z_{0h} = z_{0m} \exp \left\{ -\frac{\kappa}{P_r} \left(4\sqrt{R_0} - 3.2 \right) \right\}, \quad (12)$$

$$z_{0q} = z_{0m} \exp \left\{ -\frac{\kappa}{S_c} \left(4\sqrt{R_0} - 4.2 \right) \right\}, \quad (13)$$

$$R_0 = \max \left(0.1, \frac{z_{0m} u_*}{\nu} \right), \quad (14)$$

where κ is the von Karman constant, R_0 is the roughness Reynolds number, $P_r = 0.713$ is the neutral Prandtl number for air, and $S_c = 0.66$ is the neutral Schmidt number for water in air. Roughness lengths are allowed to be no less than 10^{-5} m .

The fraction of shortwave radiation absorbed at the surface, β , depends on the lake state. If resolved snow layers are present, then β is set equal to the absorption fraction predicted by the snow-optics submodel for the top snow layer. Otherwise, β is set equal to the near infrared fraction of the shortwave radiation reaching the surface. The remainder of the shortwave radiation fraction ($1 - \beta$) is absorbed in the lake body or soil as described in Section 5.5.

4.3 Sketch of order followed in “SLakeFluxes”

At the beginning of the timestep, the fetch, surface absorption fraction, heat of vaporization (e.g. vaporization or sublimation), and top layer thermal properties are set based on the model state at the previous timestep. Then, as in CLM4, the Saturated vapor pressure, specific humidity and their derivatives at lake surface are evaluated. The potential temperature and virtual potential temperature are evaluated. The surface roughnesses are evaluated based on

u_* from the previous timestep. The convective velocity and Monin-Obukhov Length are initialized. Then, the following loop is iterated 4 times: friction velocity, potential temperature scale, humidity scale, aerodynamic resistances, ground temperature, heat of vaporization, sensible heat flux, latent heat flux, saturated specific humidity at the surface, virtual potential temperature scale, convective velocity Monin-Obukhov Length, and surface roughnesses.

Following this loop, the ground temperature is corrected in the three cases discussed above, and the heat fluxes re-evaluated. The momentum fluxes are calculated and the ground heat flux G is evaluated as the energy flux remainder, which will be used as the top boundary condition for the sub-surface temperature calculation.

5 Lake Temperature

The module “SLakeTemperature” calculates the temperature of the full column. The column consists of between 0 and n_{sno} snow layers above the lake, n_{lak} lake layers, and n_{gnd} soil layers below the lake, comprised of n_{soi} soil layers and $n_{gnd} - n_{soi}$ bedrock layers beneath. The default values are $n_{sno} = 5$; $n_{lak} = 10$ for global runs and $n_{lak} = 25$ for site runs; and $n_{soi} = 10$, $n_{gnd} = 15$. The Crank-Nicholson Method with Tridiagonal matrix solution is used to solve simultaneously for the whole column temperature, with the appropriate thermal conductivities defined at the interfaces between layers (being careful at the interfaces between the lake and snow and between the lake and soil), and the heat capacities defined at the node (the midpoint for snow and lake layers, but not necessarily for soil, as in CLM4) of each layer.

Thermal conductivities are based on based on wind-driven eddies for unfrozen lakes based on Hostetler and Bartlein (1990), additional diffusivity due to unresolved 3D processes (Fang and Stefan, 1996), and molecular diffusion. Lake water is now to freeze by an arbitrary fraction for each layer, which releases latent heat and changes thermal properties. Convective mixing occurs for all lakes, even if frozen. If there are resolved snow layers, radiation transfer is predicted by the CLM4 snow-optics submodel, and the remaining radiation penetrating the bottom snow layer is absorbed in the top layer of lake ice; conversely, if there are no snow layers, the solar radiation penetrating the bottom lake layer is absorbed in the top soil layer. Finally, the lakes have variable depth, and all physics is assumed valid for arbitrary depth.

5.1 Boundary Conditions

The top boundary condition is a heat flux passed from SLakeFluxes and treated as constant within this module (except for resolving small energy imbalances during the phase change and convection processes). The heat flux G entering the lake from the ground surface is given by the flux remainder

$$G = \beta S_g - L_g - H_g - \lambda E_g \quad (15)$$

where β is defined in 4.2, S_g is the net ground solar radiation absorbed $S_g = (1 - a)S \downarrow$ (a is the albedo and $S \downarrow$ is the downward shortwave radiation) calculated in “SurfaceRadiation”, L_g is the net longwave flux leaving the surface, H_g is the net sensible heat flux leaving the surface, and λE_g is the net latent heat flux leaving the surface.

The bottom boundary condition is a zero flux condition imposed at the bottom of the deepest bedrock layer.

5.2 Lake Density

The density of each layer is a mass-weighted average over the water and ice fraction for each layer:

$$\rho = (1 - I)\rho_{wat} + I\rho_{ice} \quad (16)$$

where I is the ice mass fraction of the lake layer, ρ_{wat} is the temperature-dependent water density parametrization of Hostetler and Bartlein (1990) that is used in CLM4, and ρ_{ice} is the temperature-independent ice density used elsewhere in CLM.

Note that the density only will be used to determine the dynamical stability of the lake (both for eddy diffusion and convection). The thicknesses of the lake layers do not change as the lake freezes, but rather ice + water mass is constant at all times for each lake layer, and the thermal properties are adjusted to account for the fact that the ice would actually be slightly thicker. Consequently, ρ as defined here will not be equal to the lake layer mass divided by its thickness in general.

5.3 Thermal Conductivities

Lakes have an eddy diffusivity κ_e that is the sum of wind-driven eddies K_w (Hostetler and Bartlein, 1990) and enhanced diffusion due to unresolved 3D processes K_{ed} (Fang and Stefan, 1996). For unfrozen lakes ($T_g > T_f$), K_w is defined for each lake layer as in the existing CLM4, except for the following correction: the existing CLM4 contains an error in the sign of the Brunt-Väisälä frequency, since the Hostetler and Bartlein (1990) z-coordinate is positive upwards, but here it is positive downwards. $K_w = 0$ for frozen lakes. Enhanced diffusion for lake water is given by

$$K_{ed} = 1.04 \times 10^{-8} (n^2)^{-0.43}, \quad (17)$$

where K_{ed} is in m^2/s^2 , and $n > 0.0087\text{s}^{-1}$ is the Brunt-Väisälä frequency. While the liquid portion of frozen lakes undergo enhanced diffusion, $K_{ed} = 0$ for ice. The eddy mixing strength κ_e can be increased by an arbitrary factor at compile time; factors of 10-100 yielded improvements for large, deep lakes. The model also has a compile-time option to impose this increase for lakes deeper than 20 m only.

The thermal conductivity of unfrozen portions of the lake τ_{liq} is given by

$$\tau_{liq} = (\kappa_e + \kappa_m) C_{liq} \rho_{liq} \quad (18)$$

where κ_m is the molecular diffusivity of water, c_{liq} is the specific heat of water per unit mass, and ρ_{liq} is the constant approximate density of water (not the temperature-dependent density used above). For lakes containing ice, we must average the thermal conductivity over the ice and water mass fractions. Since the water and ice will more often be found in a vertically stacked configuration than a slush or a horizontally interspersed configuration, the resistances of the ice and water add, just as for the calculation of the effective thermal conductivity at layer interfaces (below). Thus, the thermal conductivity will be given by

$$\tau = \frac{\tau_{ice,eff}\tau_{liq}}{\tau_{liq}I + \tau_{ice,eff}(1 - I)} \quad (19)$$

where $\tau_{ice,eff}$ is the molecular conductivity of conductivity of ice *corrected* for the fact that the lake layers are not changing in thickness when ice forms:

$$\tau_{ice,eff} = \tau_{ice} \frac{\rho_{ice}}{\rho_{liq}} \quad (20)$$

where τ_{ice} is the standard molecular conductivity of ice, ρ_{ice} is the constant density of ice, and ρ_{liq} is the constant approximate density of water.

For soil, snow, and bedrock layers, thermal conductivity is calculated as in CLM4 for the vegetated landunit, except that frost heaving is allowed in the soil. Currently, the excess ice caused by frost heave will only be $\frac{\rho_{liq}}{\rho_{ice}}$, but future implementations will allow larger quantities of excess ice to be initialized in permafrost regions. The volume of water and ice associated with the layer, θ (volume per nominal soil volume), is

$$\theta = \frac{1}{\Delta z} \left(\frac{w_{ice}}{\rho_{ice}} + \frac{w_{liq}}{\rho_{liq}} \right), \quad (21)$$

where w_{liq} and w_{ice} are the mass per unit area of water and ice in the soil layer, respectively, and Δz is the thickness of the soil layer. In the case of excess ice, $\theta > \theta_{sat}$, the pore fraction of the soil layer (in the absence of excess ice). In this case, the layer conductivity τ calculated in 13 will be adjusted to

$$\tau_x = \frac{\tau + X\tau_{ice}}{(1 + X)^2} \quad (22)$$

$$X = \frac{\theta}{\theta_{sat}} - 1. \quad (23)$$

This form corrects for the increase in physical thickness of the layer relative to its nominal thickness, and it assumes that resistances are in parallel for the soil layer and the excess ice, which will be valid if the ice tends to be found in the form of heterogeneous vertical wedges rather than homogeneous horizontal lenses. The bedrock layers are assumed to be dry and impervious, with heat capacities and thermal conductivities as in CLM4.

The thermal conductivities at the layer interfaces are calculated assuming spatially constant flux from the node of one layer to the next, with additive

resistances. Thus, the conductivity t_j at the interface below layer j is given by

$$t_j = \frac{\tau_j \tau_{j+1} (z_{j+1} - z_j)}{\tau_j (z_{j+1} - z_{i,j}) + \tau_{j+1} (z_{i,j} - z_j)} \quad (24)$$

where τ_j is the thermal conductivity at the node of layer j , z_j is the depth of layer j , and $z_{i,j}$ is the depth of the interface below layer j .

5.4 Heat Capacities

The total heat capacity for each lake layer (in units of energy per unit area), c_v is determined by the mass-weighted average over the heat capacities for the water and ice fractions:

$$c_v = \Delta z \rho_{liq} [c_{liq}(1 - I) + c_{ice}I] \quad (25)$$

where Δz is the width of the layer. Notice that the density of water is used for both ice and water fractions. This is because the width of the layer is fixed and the ice fraction I is a mass fraction, so the mass of ice in the layer remains $I \rho_{liq} \Delta z$.

The total heat capacity for each soil, snow, and bedrock layer (in units of energy per unit area) is determined as in CLM4 for vegetated landunits, as the mass-weighted average over the heat capacities for the water, ice, and dry soil fractions.

5.5 Absorption of Solar Radiation in the Lake

The absorption of solar radiation in the lake is treated similarly to that in existing CLM4 lakes, with several modifications. First, if there are no resolved snow layers, the surface absorption fraction β is set according to the near infrared fraction, as discussed in 4.2, and the light extinction coefficient η varies between lake columns as described below. (The surface layer thickness z_a is maintained at 0.6 m.) Second, if there are resolved snow layers, then the snow-optics submodel is used to calculate the snow layer absorption (except for the absorption predicted for the top layer, assigned to the surface energy budget), with the remainder penetrating snow layers absorbed in the top lake body ice layer.

For unfrozen lakes, the solar radiation ϕ remaining at depth $z > z_a$ into the lake body, where $z_a = 0.6$ m is the base of the surface absorption layer, is given by

$$\phi = (1 - \beta) S_g \exp[-\eta(z - z_a)]. \quad (26)$$

For all lake body layers layers, the flux absorbed by the layer j , $\phi_{j-\frac{1}{2}} - \phi_{j+\frac{1}{2}}$ is given by

$$\phi_{j-\frac{1}{2}} - \phi_{j+\frac{1}{2}} = 1 - \beta S_g (\exp[-\eta(z_j - \frac{1}{2}\Delta z_j - z_a)] - \exp[-\eta(z_j + \frac{1}{2}\Delta z_j - z_a)]). \quad (27)$$

The radiation within the surface layer ($z < z_a$) is considered fixed at $(1 - \beta S_g)$; consequently, the arguments of the exponential terms above are constrained to be non-negative. (Thus, a lake layer completely above z_a will receive no direct solar flux, and a layer straddling z_a will only begin absorbing at z_a .) For the bottom layer $j = n_{lake}$, the exiting radiation is

$$\phi_{n_{lake} + \frac{1}{2}} = 1 - \beta S_g (\exp[-\eta(z_{n_{lake}} + \frac{1}{2}\Delta z_{n_{lake}} - z_a)]). \quad (28)$$

This flux is absorbed in the top soil layer. For frozen lakes, as in the existing CLM4 lake model, $(1 - \beta)S_g$ is absorbed in the top lake layer, and zero in other layers.

The light extinction coefficient η , if not provided as external data, is a function of depth, based on regression of observed Secchi depths for a database of 88 small glacial-basin lakes (Håkanson, 1995). The parameterization is given by

$$\eta = 1.1925D^{-0.424} \quad (29)$$

where D is the lake depth and η is in m^{-1} . This is based on Håkanson's Table 4D, using the standard Poole-Atkins expression relating Secchi Depth z_s to η : $\eta = 1.7/z_s$ for z_s in m.

5.6 Heat Diffusion & Tridiagonal Solution

The solution method for thermal diffusion is an extension of that in the existing CLM4 lake model. The full column consists of 25-40 levels: 0-5 snow layers, 10-25 lake layers, 10 soil layers, and 5 bedrock layers. 10 lake layers, below 0-5 snow layers. To allow seamless transition between soil, snow, and lake layers, heat capacities and thermal conductivities are used rather than the combined diffusivity $\kappa_m + \kappa_e$ used in BiogeophysicsLake. Thus, the governing equation is

$$c'_v \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left(\tau \frac{\partial T}{\partial z} \right) + \frac{d\phi}{dz} \quad (30)$$

where τ is the thermal conductivity, c'_v is the heat capacity (per unit volume), and ϕ is the radiation at depth z .

The discretized energy balance for layer j is

$$\frac{c_{v,j}}{\Delta t} (T_j^{n+1} - T_j^n) = F_{j-1} - F_j + \phi_j \quad (31)$$

where superscripts n and $n + 1$ indicate values at the beginning and end of the time step, respectively, Δt is the timestep, $c_{v,j}$ is the total heat capacity of the layer (in energy per unit area), F_j is the flux exiting the bottom of layer j , and ϕ_j is the solar flux absorbed in layer j .

This is solved using the Crank-Nicholson Method, resulting in a tridiagonal system of equations

$$r_j = a_j T_{j-1}^{n+1} + b_j T_j^{n+1} + c_j T_{j+1}^{n+1} \quad (32)$$

where

$$a_j = -0.5 \frac{\Delta t}{c_{v,j}} \frac{\partial F_{j-1}}{\partial T_{j-1}^n}, \quad (33)$$

$$b_j = 1 + 0.5 \frac{\Delta t}{c_{v,j}} \left(\frac{\partial F_{j-1}}{\partial T_{j-1}^n} + \frac{\partial F_j}{\partial T_j^n} \right), \quad (34)$$

$$c_j = -0.5 \frac{\partial F_j}{\partial T_j^n}, \quad (35)$$

$$r_j = T_j^n + 0.5 \frac{\Delta t}{c_{v,j}} (F_{j-1} - F_j) + \frac{\Delta t}{c_{v,j}} \phi_j. \quad (36)$$

For the top layer, $F_{j-1} = 2G$, where G is the ground heat flux defined in Eq. 9. (The factor of 2 is needed to cancel the Crank-Nicholson factor of $\frac{1}{2}$.) For the bottom layer, $F_j = 0$. Otherwise,

$$F_j = \frac{t_j}{z_{j+1} - z_j} (T_j^n - T_{j+1}^n) \quad (37)$$

where t_j is the thermal conductivity at the interface below layer j as defined in Eq. 16.

5.7 Phase Change

Phase change in the lake, snow, and soil is done similarly to that done for the soil and snow in CLM4 for vegetated landunits. After the heat diffusion is calculated, phase change occurs in a given layer if the temperature is below freezing and liquid water remains, or if the temperature is above freezing and ice remains.

If melting occurs, the available energy for melting, Q_{avail} , is computed as

$$Q_{avail} = (T - T_f) c_v \quad (38)$$

where T is the temperature of the layer after heat diffusion, T_f is the freezing point of water, and c_v is the heat capacity of the layer as determined in 5.4. The mass of melt in the layer, M , is given by

$$M = \min \left(M_{ice}, \frac{Q_{avail}}{Q_{fus}} \right) \quad (39)$$

where M_{ice} is the mass of ice in the layer, equal to $I \rho_{liq} \Delta z$ for a lake layer or simply the state variable $w_{ice} = \text{"h2osoi.ice"}$ for a soil / snow layer, and Q_{fus} is the heat of freezing of water per unit mass. The heat remainder, Q_{rem} is given by

$$Q_{rem} = Q_{avail} - M Q_{fus}. \quad (40)$$

Finally, the mass of ice in the layer M_{ice} is adjusted downwards by M , and the temperature T of the layer is adjusted to

$$T = T_f + \frac{Q_{rem}}{c_v}. \quad (41)$$

The heat capacity in the above is corrected by $M(c_{liq} - c_{ice})$.

If freezing occurs, Q_{avail} is again given by Eq. 30, but will be negative. The melt, M , also negative, is given by

$$M = \max\left(-M_{liq}, \frac{Q_{avail}}{Q_{fus}}\right) \quad (42)$$

where M_{liq} is $(1 - I)\rho_{liq}\Delta z$ for lakes or simply $w_{liq} = \text{“h2osoi.liq”}$ for a soil / snow layer. The heat remainder, Q_{rem} , is also given by Eq. 32 and will be negative or zero. Finally, the mass of water in the layer M_{liq} is adjusted downwards by $-M$ and the temperature is reset according to Eq. 33.

In the presence of nonzero snow depth without resolved snow layers over an unfrozen top lake layer, the available energy in the lake layer $(T_1 - T_f)c_{v,1}$ (where T_1 and $c_{v,1}$ are the temperature and heat capacity, respectively, of the top lake layer) is used to melt the snow. Similar to above, the snowwater W_{sno} is either completely melted and the remainder of heat returned to the top lake layer, or the available heat is exhausted and the top lake layer is set to freezing. The snow thickness z_{sno} is adjusted downwards in proportion to the amount of melt, $\frac{M}{W_{sno}}$, maintaining constant density.

5.8 Convective Mixing

Convective mixing is calculated in all lakes after the phase change calculation, when there is an unstable density profile, e.g. when lake layer j is more dense than the one below it, where the density is determined as in Eq. 10. Mixing also occurs when there is ice present in a layer that is below a layer which is not completely frozen. When this occurs, these two lake layers and all those above mix. In the existing CLM4 lake model, average temperature is conserved. However, there now may be ice fractions present in the mixing layers. Instead of average temperature, the average ice fraction and the total heat content are conserved, where the total heat content Q is given by

$$Q = \sum_{i=1}^{j+1} \Delta z_i \rho_{liq} (T_i - T_f) [(1 - I_i)c_{liq} + I_i c_{ice}] \quad (43)$$

where i is the lake layer, T_i is its temperature, j is the layer of heavier density than the one below that triggered the mixing, and c_{liq} and c_{ice} are the heat capacities of water and ice in energy per unit mass. The difference between the layer temperature and the freezing temperature is used to avoid a spurious apparent change in enthalpy when a layer changes ice fraction due to the difference between c_{liq} and c_{ice} .

Once the average ice fraction I_{av} is calculated from

$$I_{av} = \frac{\sum_{i=1}^{j+1} I_i \Delta z_i}{Z_{j+1}} \quad (44)$$

where

$$Z_{j+1} = \sum_{i=1}^{j+1} \Delta z_i, \quad (45)$$

the temperatures are calculated. A separate temperature is calculated for the frozen (T_{froz}) and unfrozen (T_{unfr}) fractions of the mixed layers. If the total heat content Q is positive (e.g. some layers will be above freezing), then the extra heat is all assigned to the unfrozen layers, while the fully frozen layers are kept at freezing. Conversely, if $Q < 0$, the heat deficit will all be assigned to the ice, and the liquid layers will be kept at freezing. For the layer that contains both ice and liquid (if present), a weighted average temperature will have to be calculated, being careful to weight by the enthalpy fractions and not the simple mass fractions, so that the total heat content is conserved.

If $Q > 0$, then $T_{froz} = T_f$, and T_{unfr} is calculated from

$$T_{unfr} = \frac{Q}{\rho_{liq} Z_{j+1} [(1 - I_{av}) c_{liq}]} + T_f. \quad (46)$$

If $Q < 0$, then $T_{unfr} = T_f$, and T_{froz} is calculated from

$$T_{froz} = \frac{Q}{\rho_{liq} Z_{j+1} [I_{av} c_{ice}]} + T_f. \quad (47)$$

The ice is lumped together at the top. Indeed, one advantage of mixing lakes containing ice is to maintain a reasonable vertical profile of ice content. For each lake layer i from 1 to $j + 1$, the ice fraction and temperature are set as follows, where $Z_i = \sum_{m=1}^{i-1} \Delta z_m$:

1. If $Z_i + \Delta z_i \leq Z_{j+1} I_{av}$, then $I_i = 1$ and $T_i = T_{froz}$.
2. Otherwise, if $Z_i < Z_{j+1} I_{av}$, then the layer will contain both ice and water. The ice fraction is given by

$$I_i = \frac{Z_{j+1} I_{av} - Z_i}{\Delta z_i}. \quad (48)$$

The temperature is set to conserve the desired heat content that would be present if the layer could be allowed to have two temperatures:

$$Q_i = \Delta z_i [T_{froz} I_i c_{ice} + T_{unfr} (1 - I_i) c_{wat}]. \quad (49)$$

Dividing by the true heat capacity of the layer $\Delta z_i [I_i c_{ice} + (1 - I_i) c_{liq}]$ yields the desired weighted average temperature:

$$T_i = \frac{T_{froz} I_i c_{ice} + T_{unfr} (1 - I_i) c_{wat}}{I_i c_{ice} + (1 - I_i) c_{liq}}. \quad (50)$$

3. Otherwise, $I_i = 0$ and $T_i = T_{unfr}$.

5.9 Energy Conservation

Three checks are made for energy conservation (two of which are designed mainly for development and require a compile flag to be activated): once to check the Tridiagonal solution, once after phase change and before convective mixing, and then finally after convective mixing. The first two checks are meant to be precise and report any errors in energy conservation exceeding a small computational threshold (e.g. $10^{-7} \frac{\text{W}}{\text{m}^2}$). The change in energy content for the entire column Q_{tot} is given by

$$Q_{tot} = L + \sum_{j=j_{top}}^{n_{lak}+n_{gnd}} (T_j - T_{j,bef}) c_{v,j} \quad (51)$$

where L is the latent heat absorbed (or evolved) by the column during the phase change step, j_{top} is the index of the top layer (1 minus the number of snow layers), n_{lak} is the number of lake layers, n_{gnd} is the number of soil and bedrock layers, T_j is the temperature of layer j after diffusion and phase change, and $T_{j,bef}$ is the temperature at the beginning of the timestep. Then, the error flux is reported as

$$\frac{Q_{tot}}{\Delta t} - G - (1 - \beta)S_g. \quad (52)$$

During phase change, the heat capacities of the layers may change, so this is split into two steps. First, the equations are evaluated before and after the Tridiagonal solution, without the phase change, and with constant c_v . Second, the equations are re-evaluated before and after phase change, allowing the c_v to change, with $G = 0$. The temperature difference from freezing $T_j - T_f$ is used rather than the absolute temperature to prevent apparent spurious changes in enthalpy because of the difference between c_{liq} and c_{ice} . (Otherwise, it would seem that the enthalpy would change by $MT_f(c_{liq} - c_{ice})$ as M ice is melted at the freezing point.)

After the convective mixing, the energy balance is checked again. The discrepancy will include computational errors from the convective mixing, which are likely to be the largest in the module, since by far the most vigorous thermal transport will occur in this step. This discrepancy is reconciled with the sensible heat flux, as long as it is less than $0.1 \frac{\text{W}}{\text{m}^2}$; otherwise the error will be sent to the ‘‘BalanceCheck’’ module where the model will be aborted. At the beginning of the timestep, the enthalpy content H_{in} has been summed as

$$H_{lak} = \sum_{j=1}^{n_{lak}} [c_{v,j}(T_j - T_f) + Q_{fus}\rho_{liq}\Delta z_j(1 - I)]. \quad (53)$$

The temperature difference from freezing is again used to reconcile the difference in apparent enthalpy between water and ice at freezing, if the enthalpy is naively summed merely as $(c_{liq}M_{liq} + c_{ice}M_{ice})T + M_{liq}Q_{fus}$. Added to this is

$$H_{soisno} = \sum_{j=j_{top}}^{n_{soi}} [c_{v,j}T_j + Q_{fus}w_{liq}]. \quad (54)$$

$H_{in} = H_{lak} + H_{soisno}$. Finally, because a small amount of snow (without snow layers) may be melted during the phase change, terms are added so that the latent heat absorbed due to this snow melt is accounted for.

After the convective mixing, the heat capacities are re-evaluated, and the final enthalpy is given similarly by $H_{fin} = H_{lak} + H_{soisno}$, but with updated temperatures, heat capacities, and ice fractions. The error flux E_{soi} is evaluated as

$$E_{soi} = \frac{H_{fin} - H_{in}}{\Delta t} - G - (1 - \beta)S_g. \quad (55)$$

If $|E_{soi}| < 0.1 \frac{\text{W}}{\text{m}^2}$, it is subtracted from the sensible heat flux and added to G .

6 Lake Hydrology

6.1 Overview

The lake hydrology is found in the module ‘‘SLakeHydrology’’. Full hydrology of snow layers is done as in non-lake columns in CLM4. However, there is no infiltration, and the water budget is balanced with q_{rgwl} , a generalized runoff term for special (i.e. non-vegetated) landunits that may be negative. As in CLM4, lake water mass is kept constant. The soil is simply maintained at volumetric saturation if the melting of ice frees up pore space. Likewise, if the liquid water volume in the soil exceeds pore capacity, it is reduced to pore capacity. This is consistent with the possibility of initializing some soil layers with excess ice in future implementations.

If snow layers are found in the model over an unfrozen lake, and the top layer of the lake is capable of absorbing the latent heat without going below freezing, the snow-water is runoff and the latent heat is subtracted from the lake. To preserve numerical stability in the lake model, two changes are made for snow layers. First, dew or frost is not allowed to be absorbed by a top snow layer which has become completely melted during the timestep. Second, because of occasional instabilities occurring during model testing due to the interaction of the surface flux and phase change solution (and that these coupled equations are not iterated several times for each timestep to make sure that they are consistent), resolved snow layers must be a minimum of $s_{min} = 4$ cm thick rather than 1 cm.

6.2 Water Balance

The total water balance of the system is given by

$$\Delta W_{sno} + \sum_{j=1}^{n_{soi}} (\Delta w_{liq,j} + \Delta w_{ice,j}) = (q_{rain} + q_{sno} - E_g - q_{rgwl} - q_{sci})\Delta t \quad (56)$$

where W_{sno} is the total mass of snow (both liquid and ice), $w_{liq,j}$ and $w_{ice,j}$ are the masses of water phases in soil layer j , q_{rain} and q_{sno} are the precipitation

forcing from the atmosphere in units of mass per unit area per unit time, q_{sci} is the ice runoff associated with snow-capping (calculated as in CLM4) and E_g is the ground evaporation.

6.3 Snow

The snow hydrology is nearly identical to that in CLM4 for non-lake landunits. Treatment of snow density, snow water, snow capping, snow compaction, and dissolved deposited aerosol species should be identical. Snow layer initialization, combination, and division are adjusted so that resolved snow layers are at least $s_{min} = 4$ cm thick rather than 1 cm (see below). Solid precipitation is added immediately to the snow, while liquid precipitation is added to snow layers, if they exist, after accounting for dew, frost, and sublimation. If there is enough snow for resolved snow layers ($z_{sno} > s_{min}$) but none present, they are initialized before the dew, frost, and sublimation occurs. In the case of snow found over an unfrozen lake, see 6.7 below.

6.4 Modifications to Snow Layer Logic

A model constant is added, $z_{lsa} = s_{min} - 1\text{cm} = 3\text{cm}$, to thicken the minimum resolved snow layer thickness for lake columns as compared to non-lake columns in the existing CLM4. (The existing CLM4 lake model does not have any resolved snow layers.) When “SnowHydrologyMod:CombineSnowLayers” is called for lake columns, the minimum layer thicknesses (as a function of the number of snow layers present) all are increased by z_{lsa} . Likewise, there is a new routine “SnowHydrologyMod:CombineSnowLayersLake” that applies only over lake columns. Here, as in the original routine, there are a series of rules for dividing snow layers as a function of the number of layers and the layer thicknesses. There are two types of operations: (a) subdividing layers in half, and (b) shifting some volume from higher layers to lower layers (without increasing the layer number). For subdivisions of type (a), the thickness thresholds triggering subdivision are increased by $2z_{lsa}$ for lakes. For shifts of type (b), the thickness thresholds triggering the shifts are increased by z_{lsa} . At the end of “SnowHydrologyMod:CombineSnowLayersLake”, a snow ice and liquid balance check are performed to make sure alterations to these rules have not caused an error in the total snow mass.

6.5 Precipitation, Dew, Frost, & Sublimation

All precipitation reaches the ground, as there is no vegetated fraction. If there are snow layers, “evaporation” (e.g. evaporation, dew, frost, and sublimation) is treated as over non-lake landunits in CLM4, except that the allowed evaporation from the ground is unlimited (though the top snow layer cannot lose more water mass than it contains).

If there are no snow layers, then “evaporation” occurs similarly to that in the existing CLM4 lake model (in “HydrologyLake”), except that the snow may still

have a variable density. If the ground evaporation E_g is positive, then $q_{sub,sno}$, the sublimation from the snow, is given by

$$q_{sub,sno} = \min\left(E_g, \frac{W_{sno}}{\Delta t}\right). \quad (57)$$

If $E_g < 0$, $T_g \leq T_f$, and the top snow layer is not unfrozen, then $q_{frost} = |E_g|$, where q_{frost} is the rate of frost production. If $E_g < 0$ but the top snow layer has completely thawed during the Phase Change step of the Lake Temperature solution, then frost (or dew) is not allowed to accumulate, to insure that the layer is eliminated by the Snow Hydrology code. The snowpack is updated for frost and sublimation:

$$W_{sno} = W_{sno} + \Delta t(q_{frost} - q_{sub,sno}) \quad (58)$$

where the q_{frost} term is only added if the snow is not currently capped at that column. The snow thickness z_{sno} is adjusted by the same proportion as the snow water (e.g. maintaining the same density), unless there was no snow before adding the frost, in which case the density is assumed to be $250 \frac{\text{kg}}{\text{m}^3}$.

6.6 Soil Hydrology

If the water and ice soil volume in a soil layer θ (Eq. 15) is less than the pore volume saturation θ_{sat} (as may occur when ice melts), then the liquid water mass is adjusted to

$$w_{liq} = \left(\theta_{sat}\Delta z - \frac{w_{ice}}{\rho_{ice}}\right) \rho_{liq}. \quad (59)$$

Otherwise, if excess ice is melting and $w_{liq} > \theta\rho_{liq}\Delta z$, then the water in the layer is reset to

$$w_{liq} = \theta\rho_{liq}\Delta z. \quad (60)$$

This allows excess ice to be initialized (and begin to be lost only after the pore ice is melted, which is realistic if the excess ice is found in heterogeneous chunks) but irreversibly lost when melt occurs.

6.7 Snow Elimination

Snow layers may be found in the model above a lake with an unfrozen top layer. In reality, the snow would fall into the lake and melt or turn to ice. To attempt to simulate this, the top lake layer will give up heat to melt the snow if it has excess above freezing. If the top lake layer has sufficient heat to melt the snow without freezing, then this will be done. If not, the top lake layer will undergo freezing, but only if it will not freeze completely. Otherwise, the modeled snow layers will persist and continue to melt by diffusion.

If the top layer is completely unfrozen and there are snow layers, then the snow ice $W_{snowice}$ is calculated by summing the ice in each snow layer, and the

enthalpy deficit H_{def} of the snow layers below freezing is calculated as

$$H_{def} = \sum_{j=-n_{sno}+1}^0 (w_{ice}c_{ice} + w_{liq}c_{liq})(T_f - T_j). \quad (61)$$

Then, H_{melt} is set as

$$H_{melt} = H_{def} + W_{snowice}Q_{fus}. \quad (62)$$

The heat remainder of the top lake layer, Q_{rem} , is

$$Q_{rem} = (T_1 - T_f)c_{liq}\rho_{liq}\Delta z_1 - H_{melt} \quad (63)$$

where T_1 and Δz_1 are the temperature and thickness of the top lake layer. If the top lake layer can provide the heat without totally freezing, e.g. if $Q_{rem} + \rho_{liq}\Delta z_1 Q_{fus} > 0$, then the snow is eliminated. If $Q_{rem} > 0$, then

$$T_1 = T_1 - \frac{Q_{rem}}{c_{liq}\rho_{liq}\Delta z_1}. \quad (64)$$

Otherwise, $T_1 = T_f$, and the top layer lake ice fraction I_1 is set to

$$I_1 = \frac{-Q_{rem}}{\rho_{liq}\Delta z_1 Q_{fus}}. \quad (65)$$